Technical Notes

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Momentum Integral Method at the Beginning of Crossflow Development

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Nomenclature

a	= Stock's streamwise flow parameter
$a_1,a_2,a_3,$ $b_1,b_2,b_3,$ h_1,h_2,q	= metric coefficients
b	= Stock's crossflow parameter
c	= scale factor for crossflows
M	= Mach number
u	= x component of velocity
v	= y component of velocity
x,y,z	= coordinate directions
α	= angle between freestream direction and local x direction
β	= angle between freestream direction and local skin friction
ϵ	= boundary-layer thickness of reference profiles
η	= nondimensional distance normal to the surface
θ	= momentum thickness in streamline coordinates
Φ	= Stock's momentum thicknesses in the nonorthogonal coordinate system

Subscripts

e	= resultant freestream quantity
R	= reference quantity
x	= x component
y	= y component
δ	= edge of the boundary layer
$,(\ )$	= derivative with respect to the accompanying quantity

Introduction

INTEGRAL methods have many computational advantages over finite difference methods requiring large resources. This feature makes integral methods attractive in the study of flowfields with distinctively different flow regimes, such as multielement airfoil studies. There has been a considerable amount of work done in the study of three-dimensional flowfields using integral methods. The work of Prandtl,[1] Striuminskii,[2] Jones,[3] Sears,[4] and Cooke[5] are based on the principle of the independence of crossflow velocities. The method adopted by Cumpsty and Head,[6] using Mager's[7] crossflow profiles and Thompson's[8] streamwise profiles, predicts the flowfield efficiently when the crossflow is in one direction throughout the boundary layer. But in situations where the velocity direction changes, crossover profiles will have to be taken into account. Stock[9] has proposed a general way of computing three-dimensional laminar flowfields with crossover profiles. This Note modifies Stock's method in order to make it adaptable to situations in which the original method showed instability. Stock's method uses a nonorthogonal coordinate system similar to that described by Squire.[10] The equations of motion are derived in the same manner as that used by Myring[11] and Smith[12] in their work on turbulent boundary layers. Stock uses the "moment-of-momentum" equations in addition to the momentum and continuity equations. The streamwise velocities are parameterized with respect to the Falkner-Skan profiles. The crossflow profiles are parameterized with respect to an arbitrary family of crossflow profiles, which has crossover profiles as its members. This choice of profiles gives very accurate results when studying cases in which the three-dimensional boundary layers are well developed. However, in the calculation of boundary layers over aircraft wings special situations occur. It is not uncommon in computational approaches that one has to use the two-dimensional approximation for one region and then switch to three-dimensional computation for the subsequent region. Stock's method, although well formulated in the regions of well-developed three-dimensional flows, showed instability at the regions of the switchover. The present effort is to isolate the problem at this transition phase of numerical computation. It is shown in this Note that the crossflow profiles should be chosen in such a way as to satisfy a compatibility condition. Smith has shown that the presence of characteristic lines has to be accounted for while studying regions with the small crossflows normal to the external streamline. The analysis of small-crossflow regions is important in the study of flowfields over aircraft wings and a proper choice of profiles is necessary in order to prescribe the correct initial values.

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Equations for Three-Dimensional Boundary Layers

A rectangular coordinate system is chosen with the x direction along the freestream direction, the y direction normal to the x direction directed toward the wing tip, and the z direction normal to the surface. Squire's representation is used for the equations of motion. The x and y momentum equations are combined with the continuity equation and are integrated from the surface to the edge of the boundary layer. The moment-of-momentum equations are derived by multiplying the momentum equations by the respective component of velocity and combining with the continuity equation. These equations are also integrated across the boundary layer. The resulting equations in terms of the various integral thicknesses are given below.

$$\frac{1}{h_1}\Phi_{11,x}+\Phi_{11}\frac{(2-M_e^2)}{h_1}\frac{U_{e,x}}{U_e}+\frac{1}{h_2}\Phi_{12,y}+\Phi_{12}\frac{(2-M_e^2)}{h_2}\frac{U_{e,y}}{U_e}$$
$$+\Delta_1\frac{1}{h_1}\frac{U_{1,x}}{U_e}+\Delta_2\frac{1}{h_2}\frac{U_{1,y}}{U_e}=\frac{C_{f_x}}{2} \tag{1}$$

$$\frac{1}{h_1}\Phi_{21,x}+\Phi_{21}\frac{2-M_e^2}{h_1}\frac{U_{e,x}}{U_e}+\frac{1}{h_2}\Phi_{22,y}+\Phi_{22}\frac{2-M_e^2}{h_2}\frac{U_{e,y}}{U_e}$$
$$+\Delta_1\frac{1}{h_1}\frac{V_{1,x}}{U_e}+\Delta_2\frac{1}{h_2}\frac{V_{1,y}}{U_e}=\frac{C_{f_y}}{2} \tag{2}$$

$$\frac{1}{h_1}\Phi_{111,x}+\Phi_{111}\frac{3-M_e^2}{h_2}\frac{U_{e,x}}{U_e}+\frac{1}{h_2}\Phi_{112,y}+\Phi_{112}\frac{3-M_e^2}{h_2}\frac{U_{e,y}}{U_e}$$
$$+2(\Delta_1-\Delta_{1i})\frac{1}{h_1}\frac{U_1}{U_e^2}U_{1,x}+2(\Phi_{21i}-\Phi_{12})\frac{1}{h_2}\frac{U_{1,y}}{U_e}=2S_x \tag{3}$$

$$\frac{1}{h_1}\Phi_{221,x}+\Phi_{221}\frac{3-M_e^2}{h_1}\frac{U_{e,x}}{U_e}+\frac{1}{h_2}\Phi_{222,y}+\Phi_{222}\frac{3-M_e^2}{h_2}\frac{U_{e,y}}{U_e}$$

$$+2(\Delta_2-\Delta_{2i})\frac{1}{h_2}\frac{V_1}{U_e^2}V_{1,y}+2(\Phi_{12,i}-\Phi_{21})\frac{1}{h_1}\frac{V_{1,x}}{U_e}=2S_y \tag{4}$$

where the integral thicknesses are those defined in the Appendix. In order to parameterize the flow profiles, the momentum thicknesses of Eqs. (1-4) are expressed in terms of momentum thicknesses of streamwise and crosswise flow quantities. Stock's approach is to parameterize these integral functions in terms of two distinct parameters, one for the streamwise flow and another for the crosswise flow. The similarity profiles of the Falkner-Skan representation are used for the streamwise functions. The transformation from the similarity coordinates to the real coordinates is obtained by computing the momentum thickness in the real coordinates and its counterpart in the similarity coordinates at the same time. The parameters chosen for representing each of the profiles are

$$a=\left.\frac{\partial u}{\partial \eta}\right|_{\eta=\text{wall}}\int_0^{\eta_\delta}U(1-U)\,\mathrm{d}\eta \tag{5}$$

The definition of crossflow profiles normal to the external streamline is very important in order to obtain a consistent solution. Stock's method gives the basic approach to be used in order to define the process of generating a family of profiles which has the crossover profile as a member. This method consists of choosing one reference profile from the Falkner-Skan profiles. This reference profile is then scaled up to have a δ value of 6.75. The crossflow profiles are found by subtracting the individual Falkner-Skan profiles from the reference profile and then scaling it down to a δ value of 1. The details of this process are explained by Stock. The parameter chosen for representing the crossflow profiles is

$$b=\int_0^1 V_R\,\mathrm{d}\eta \tag{6}$$

where

$$V=cV_R,\quad c=c(x) \tag{7}$$

The computation of integral thicknesses which involve crossproduct terms of both u and v is done assuming the same boundary-layer thickness for both crossflow and streamwise flow. It will be shown later that this assumption has to be relaxed in order to solve the equations.

Each of the unknowns in Eqs. (1-4) is expressed in terms of θ_{11}, a, b, and c using the relation

$$A_x=A_{\theta_{11}}\theta_{11,x}+A_{,a}a_{,x}+A_{,b}b_{,x}+A_{,c}c_{,x} \tag{8}$$

Thus there exist four equations for the solution of $\theta_{11,x}$, a_x, b_x, and c_x, respectively. A numerical integration can be done subsequently in order to determine θ_{11}, a, b, and c.

Consistency Condition

Stock provides a very general method for analyzing three-dimensional laminar flows where different families of velocity profiles can be fed into the system by four simultaneous equations, depending on the kind of flowfield study being undertaken. The transition from a two-dimensional flow situation to a three-dimensional flow will generate some constraints to be imposed on the family of profiles chosen for the study. The purpose of the present study is to isolate such constraints. Equations (1) and (2) can be rewritten as

$$\begin{vmatrix}\Phi_{11,R_{11}} & \Phi_{11,a} & \Phi_{11,b} & \Phi_{11,c}\\ \Phi_{21,\theta_{11}} & \Phi_{21,a} & \Phi_{21,b} & \Phi_{21,c}\\ \Phi_{111,\theta_{11}} & \Phi_{111,a} & \Phi_{111,b} & \Phi_{111,c}\\ \Phi_{221,\theta_{11}} & \Phi_{221,a} & \Phi_{221,b} & \Phi_{221,c}\end{vmatrix}\begin{vmatrix}\Phi_{11,x}\\ a_{,x}\\ b_{,x}\\ c_{,x}\end{vmatrix}=\begin{vmatrix}D_1\\ D_2\\ D_3\\ D_4\end{vmatrix} \tag{9}$$

or

$$TP=D$$

The y derivatives are included on the right side of Eq. (9). A solution to Eq. (9) exists everywhere except possibly at regions where the determinant of T vanishes. This condition exists when the crossflow is zero, i.e.,

$$c=0\quad\text{and}\quad c_{,x}\neq 0$$

At this condition, the third-column vector in the matrix T is identically zero. Physically, this corresponds to a two-dimensional flow and only two equations are required to solve the flowfield. Any attempt to use the four equations to calculate the derivatives of the flow parameters will result in numerical instability. In the flowfield study of aircraft wings, the switch from two-dimensional flow to three-dimensional flow occurs when it is possible to approximate the value of c to be close to zero and the spatial derivatives of c are significant. This in turn means that a finite value should exist for the derivative of b. A closer look at Eq. (9) provides the following analytical expression for the derivative of b

$$b_{,x}=\sum_{J=1}^{4}C_{J3}D_J/\text{DET }T$$

where C_{J3} is the cofactor of the elements of the third column of the T matrix.

One necessary condition for $b_{,x}$ to have a finite value, if it exists, is that the numerator expression vanishes if DET T is zero. This in turn gives the result that

$$\sum_{J=1}^{4}C_{J3}D_J=0$$

where C_{J3} is the cofactor of the elements of the third column of the T matrix. This in turn results in the equation

$$(\theta_{11}+\delta_1)/\theta_{21,c}=(\delta_1-\delta_{1i}+\theta_{111})/(\theta_{112,c}+\delta_{2,c}) \tag{10}$$

Equation (10) must be satisfied at the regions close to the zero crossflow situation. The profiles given by Stock satisfy this condition when using the crossover profiles. But at the start of the development of crossflow, the velocity profiles must satisfy another condition. Near the wall and for attached flow the crossflow velocities can be approximated in the following way

$$V_R|_{\text{wall}}\simeq U_R\,\text{sign}(\beta)$$

For accelerating flows the second derivative of the streamwise velocity component is negative. This in turn gives the following condition

$$\left.\frac{\delta^2(V/U_e)_R}{\partial\eta^2}\right|_{\text{wall}}\text{sign}(\beta)\leq 0 \tag{11}$$

Since both Eqs. (11) and (10) must be satisfied at the beginning of crossflow, a new set of crossflow profiles will have to be defined. This problem can be solved by assuming a different value of η_δ for the streamwise and crosswise profiles. This would alter the value of momentum thicknesses involving crossproducts of the u and v velocities. By choosing δ_u to be 8 and δ_v to be 6.75, it was possible to arrive at profiles which simultaneously satisfy both Eqs. (10) and (11). It is possible to arrive at similar sets of profiles by choosing different combinations of δ_u and δ_v.

The above condition does not provide information about how to calculate $b_{,x}$, but does provide information about one of the conditions to be satisfied in order for a unique solution for $b_{,x}$ to exist. At the location where c is the approximately zero, b, $c_{,x}$, and $c_{,y}$ are not negligible. Equation (9) can be used to obtain an expression for $c_{,x}$, viz.,

$$c_{,x} = -(\theta_{11}+\delta_1)/\theta_{21,c}(\delta_{,x}+\delta_{,y}) - c_{,y} \tag{12}$$

Equation (12) is particularly useful in the calculation of three-dimensional flow with small crossflow, where the two-dimensional equations can be used to solve for the streamline quantities.

Conclusion

The analysis shows that the method suggested by Stock can be used to solve three-dimensional laminar boundary layers by choosing a family of profiles which take into consideration the consistency relation in small crossflow regions, while the earlier approach resulted in numerical instability.

The computation of flowfield parameters at the switchover point is still not fully understood, but one condition which caused instability in the computation has been isolated.

Appendix: Definition of Integral Thicknesses

Lower case letters indicate quantities in the rectangular coordinate system and upper case letters indicate those in the streamline coordinate system. The suffix R refers to reference profiles.

$$\phi_{11} = \int_0^\delta \frac{1}{\rho_e u_e^2}\rho u(u_1-u)\delta Z \qquad \phi_{111} = \int_0^\delta \frac{1}{\rho_e u_e^3}\rho u(u_1^2-u^2)\delta Z$$

$$\phi_{12} = \int_0^\delta \frac{1}{\rho_e u_e^2}\rho v(u_1-u)\delta Z \qquad \phi_{112} = \int_0^\delta \frac{1}{\rho_e u_e^3}\rho v(u_1^2-u^2)\delta Z$$

$$\phi_{21} = \int_0^\delta \frac{1}{\rho_e u_e^2}\rho u(v_1-v)\delta Z \qquad \phi_{221} = \int_0^\delta \frac{1}{\rho_e u_e^3}\rho u(v_1^2-v^2)\delta Z$$

$$\phi_{22} = \int_0^\delta \frac{1}{\rho_e u_e^2}\rho v(v_1-v)\delta Z \qquad \phi_{222} = \int_0^\delta \frac{1}{\rho_e u_e^3}\rho v(v_1^2-v^2)\delta Z$$

$$\phi_{121} = \int_0^\delta \frac{v}{\rho_e u_e^2}(\rho_e u_1-\rho u)\delta Z \qquad \phi_{211} = \int_0^\delta \frac{u}{\rho_e u_e^2}(\rho_e v_1-\rho v)\delta Z$$

$$\Delta_1 = \int_0^\delta \frac{1}{\rho_e u_e}(\rho_e u_1-\rho u)\delta Z \qquad \Delta_{11} = \int_0^\delta \frac{1}{u_e}(u_1-u)\delta Z$$

$$\Delta_2 = \int_0^\delta \frac{1}{\rho_e u_e}(\rho_e v_1-\rho v)\delta Z \qquad \Delta_{21} = \int_0^\delta \frac{1}{u_e}(v_1-v)\delta Z$$

$$C_{fx} = 2/(\rho_e u_e)(\mu(u/u_e)_{,z})_{Z=0}$$

$$C_{fy} = 2/(\rho_e u_e)(\mu(v/u_e)_{,z})_{Z=0}$$

$$S_x = 1/(\rho_e u_e)\int_0^\delta \mu(u/u_e)_{,Z}^2\delta Z$$

$$S_y = 1/(\rho_e u_e)\int_0^\delta \mu(v/u_e)_{,Z}^2\delta Z$$

Where subscript 1 denotes conditions just external to boundary layer

$$\theta_{11} = \int_0^\delta \frac{1}{\rho_e U_e^2}\rho U(U_1-U)\delta Z \qquad \theta_{111} = \int_0^\delta \frac{1}{\rho_e U_e^3}\rho U(U_1^2-U^2)\delta Z$$

$$\theta_{12} = \int_0^\delta \frac{1}{\rho_e U_e^2}\rho V(U_1-U)\delta Z \qquad \theta_{112} = \int_0^\delta \frac{1}{\rho_e U_e^3}\rho V(U_1^2-U^2)\delta Z$$

$$\theta_{21} = \int_0^\delta \frac{1}{\rho_e U_e^2}\rho U(V_1-V)\delta Z \qquad \theta_{221} = \int_0^\delta \frac{1}{\rho_e U_e^3}\rho U(V_1^2-V^2)\delta Z$$

$$\theta_{22} = \int_0^\delta \frac{1}{\rho_e U_e^2}\rho V(V_1-V)\delta Z \qquad \theta_{222} = \int_0^\delta \frac{1}{\rho_e U_e^3}\rho V(V_1^2-V^2)\delta Z$$

$$\theta_{121} = \int_0^\delta \frac{V}{\rho_e U_e^2}(\rho_e U_1-\rho U)\delta Z \qquad \theta_{211} = \int_0^\delta \frac{U}{\rho_e U_e^2}(\rho_e V_1-\rho V)\delta Z$$

$$\delta_1 = \int_0^\delta \frac{1}{\rho_e U_e}(\rho_e U_1-\rho U)\delta Z \qquad \delta_{11} = \int_0^\delta \frac{1}{U_e}(U_1-U)\delta Z$$

$$\delta_2 = \int_0^\delta \frac{1}{\rho_e U_e}(\rho_e V_1-\rho V)\delta Z \qquad \delta_{21} = \int_0^\delta \frac{1}{U_e}(V_1-V)\delta Z$$

$$C_{fT} = 2/(\rho_e U_e)(\mu(U/U_e)_{,Z})_{Z=0}$$

$$C_{fN} = 2/(\rho_e U_e)(\mu(V/U_e)_{,Z})_{Z=0}$$

$$S_T = 1/(\rho_e U_e)\int_0^\delta \mu(U/U_e)_{,Z}^2\delta Z$$

$$S_N = 1/(\rho_e U_e)\int_0^\delta \mu(V/U_e)_{,Z}^2\delta Z$$

$$S_{TN} = 1/(\rho_e U_e)\int_0^\delta \mu(U/U_e)_{,Z}(V/U_e)_{,Z}\delta Z$$

$$K_1 = \int_0^\epsilon (1-U_R)\delta\eta \qquad K_2 = -\int_0^1 V_R\delta\eta$$

$$K_{11} = \int_0^\epsilon U_R(1-U_R)\delta\eta \qquad K_{22} = -\int_0^1 V_R^2\delta\eta$$

$$K_{111} = \int_0^\epsilon U_R(1-U_R)\delta\eta \qquad K_{222} = -\int_0^1 V_R^3\delta\eta$$

$$f''_W = (U_{R,\eta})_W \qquad g''_W = (V_{R,\eta})_W$$

$$L_{11} = \int_0^\epsilon (U_{R,\eta})^2\delta\eta \qquad L_{22} = \int_0^1 (V_{R,\eta})^2\delta\eta$$

$$K_{12} = -\int_0^\epsilon U_R V_R\delta\eta \qquad K_{112} = -\int_0^\epsilon U_R^2 V_R\delta\eta$$

$$K_{221} = -\int_0^\epsilon U_R V_R^2\delta\eta \qquad L_{12} = \int_0^\epsilon U_{R,\eta}V_{R,\eta}\delta\eta$$

$$C_R = (T_W/T_e)^{0.5}(T_e+102.5)/(T_W+102.5)$$

$$m_e^2 = (\gamma-1)/2M_e^2 \qquad R_{e\theta} = \theta_{11}U_e\rho_e/\mu_e$$

$$\delta_1 = \theta_{11}(K_1/K_{11}+m_e^2(K_1/K_{11}+1))$$

$$\delta_{11} = \theta_{11}(K_1/K_{11}(1+m_e^2)+m_e^2(1-K_{111}/K_{11}))$$

$$\theta_{111} = \theta_{11}(1+m_e^2K_{111}/K_{11}), \quad C_{fT} = 2aC_R/R_{e\theta}$$

$$S_{TT} = L_{11}K_{11}C_R/R_{e\theta}, \quad \delta_2 = \theta_{11}\epsilon bC/K_{11}$$

$$\delta_{21} = \theta_{11}C(\epsilon b/K_{11}+m_e^2(\epsilon b/K_{11}-K_{111}/K_{11}))$$

$$\theta_{22} = \theta_{11}C^2\epsilon K_{11}/K_{111}, \quad \theta_{222} = \theta_{11}C^2\epsilon K_{222}/K_{11}$$

$$C_{fN}=2g''_W K_{11}C^2 C_R/R_{e\theta}/\epsilon, \quad S_{NN}=L_{22}K_{11}C^2 C_R/R_{e\theta}/\epsilon$$

$$\theta_{12}=\theta_{11}C(K_{12}/K_{11}-\epsilon b/K_{11}), \quad \theta_{121}=\theta_{11}CK_{12}/K_{11}-\delta_{21}$$

$$\theta_{112}=\theta_{11}C(K_{112}/K_{11}-\epsilon b/K_{11})$$

$$\theta_{221}=\theta_{11}C^2K_{221}/K_{11}, \quad S_{TN}=L_{12}K_{11}CC_R/R_{e\theta}$$

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Similarity Relation for Vortex-Asymmetry Onset on Slender Pointed Forebodies

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Introduction

THE subject of similarity conditions for the onset of vortex asymmetry for laterally symmetric flow (zero sideslip) over slender pointed forebodies and wings at high angles of attack at subsonic speeds has been explored from an experimental viewpoint by Keener and Chapman.[1] The subject has been further addressed by Ericsson and Reding in their comprehensive survey paper[2] on vortex-induced asymmetric loads. Ericsson and Reding emphasize the importance of forebody asymmetric vortices and observe that, for slender pointed bodies with an afterbody, the available evidence indicates that the nose-generated vortices not only are responsible for the larger asymmetric loads but also influence greatly the vortex shedding over the afterbody.

Although a number of aerodynamic schemes for attenuating or eliminating vortex-induced asymmetric loads on forebodies are known, the actual cause of the vortex-asymmetry onset phenomenon is not completely understood. There are at least two schools of thought on a possible cause for the asymmetry onset. One school proposes that a peripheral asymmetry of the viscous flow separation points precipitates the vortex asymmetry, whereas another school believes that an inviscid hydrodynamic instability due to the crowding together of the symmetric vortex pair is responsible for the asymmetry.

On the basis of some comparisons with a limited amount of experimental data, Keener and Chapman[1] show that, at high angles of attack and zero sideslip, an asymmetric moment (in roll) occurs on slender delta wings which is similar to the occurrence of an asymmetric force (side force) on slender circular forebodies. They comment that since the asymmetry in the forebody forces is known to be associated with an asymmetry in the vortex flowfield on the lee side, the asymmetry in the wing rolling moment must be associated with a similar asymmetry in the wing vortex flowfield. The fact that the angle of attack at which asymmetry occurs decreases with increasing slenderness suggested to Keener and Chapman that the cause of the vortex asymmetry is a hydrodynamic instability in the vortex flowfield resulting from the crowding together of the vortices as the apex angle is decreased. It is further suggested by the aforementioned investigators that the fact that the separation point is fixed at the leading edge for delta wings implies that an asymmetry in the separation points on a body of revolution is not necessarily an essential feature of vortex asymmetry.

It is the purpose of this Note to provide an approximate theoretical basis for the experimental observations of Keener and Chapman.[1]

Analysis

The analysis employs slender-body theory, with the various cross-flow planes used summarized in Fig. 1. Except for the $\hat{X}$ and $\bar{X}$ planes, all paired planes (denoted by the arrows on the figure) are conformally mapped onto each other. A cross section of the flow about the subject forebody at a given x station (viewed looking upstream) is shown in Fig. 1a. Body axes x, y, and z are employed, with the x axis positive in the general direction of the positive freestream U_∞. The cross-flow velocity components v and w are respectively parallel to y and z. The complex variable for the X plane is $X=y+iz$. For a complex potential defined by $W=\phi+i\Psi$, the conjugate complex velocity is given by $v-iw=\mathrm{d}W/\mathrm{d}X$. The angle of attack α is given by $\alpha=w_\infty/U_\infty$. Corresponding quantities in cross-flow planes other than the X plane are denoted by applying the overscript or superscript symbol (tilde, caret, bar, or asterisk) appropriate to the particular plane under consideration, as shown in Fig. 1. The body cross section shown in Fig. 1a is an ellipse, although any cross-sectional shape that can be mapped conformally into a circle may be analyzed. The freestream velocity U_∞, the lateral dimension $a(x)$, and the x coordinate are invariant from cross-flow plane to cross-flow plane. The latter condition requires identical planforms (not restricted to straight leading edges) for the variously transformed bodies.

The purpose of the cross-flow plane progressions shown in Fig. 1 is to relate the flow about an arbitrary laterally symmetric thick body having arbitrarily located laterally symmetric separation points s and s' in the X plane to the flow about a zero-thickness wing of a similar planform shape with separation points s and s' at the wing leading edges in the X^* plane. In contrast to the usual mapping procedure wherein the

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